ELECTRICAL CONDUCTIVITY OF A GAS IN PRESENCE OF EXTERNAL IONIZING RADIATION

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A numerical calculation is made of the current-voltage characteristic of a gas gap ionized by an external source in the case of ion–ion conductivity. The results obtained are compared with known approximate analytic solutions. The method of numerical integration of the set of equations is discussed.

The current-voltage characteristic of a gas ionized by an external source is calculated in [1,2], both allowing for and neglecting space charge for an arbitrary geometry of electric field. The attempts to obtain an exact analytic solution when space charge is taken into account run into considerable mathematical difficulties, and the authors accordingly restricted the discussion to two limiting cases: Ohm's law obeyed (voltage $V \to 0$); almost complete collection of ions ($V \to \infty$). The solutions obtained are used in [2] to derive an approximate formula for the conduction current at arbitrary voltages. The method used to extrapolate the current-voltage characteristic to intermediate voltages is not, however, unambiguous, and the accuracy of the approximate formula obtained in [2] thus stands in need of additional refinement. In the work reported in the present paper a digital computer was used to obtain a numerical solution to the appropriate set of equations, and the solution was used to determine the current-voltage characteristic for values of the collection efficiency f in the range $0.18 < f < 1$. As shown in [2], the set of differential equations in which allowance is made for space charge always has the same form for any system of electrodes with uniformly distributed surface charge (in particular, for planar, cylindrical, and spherical geometries). Accordingly, in the present paper for the sake of simplicity the current-voltage characteristic will be calculated and the computational procedure discussed on the example of a planar geometry.

The initial set of equations, allowing for the creation and recombination of positive and negative ions, has the following dimensionless form:

$$\frac{\varepsilon}{dx} = -\frac{4\pi}{\varepsilon}(j-(1+\mu)I), \qquad \frac{dI}{dx} = 1+\frac{\lambda}{\varepsilon^2}I(I-f) \tag{1}$$

$$\left(\varepsilon = \sqrt{\frac{K_+}{eqd^2}}E,\ I = \frac{I_-}{eqd},\ f = \frac{J}{eqd},\ \lambda = \frac{\alpha}{eK_-},\ \mu = \frac{K_+}{K_-}\right)$$

Here ε is the dimensionless electric field; I is the dimensionless current of negative ions; f is the collection efficiency; λ and μ are the dimensionless parameters of the problem; q is the intensity of formation of ions by the external source; J is the total current; α is the ionic recombination coefficient; x is the dimensionless coordinate, $0 \le x \le 1$ ($x = 0$ is the cathode); and K_+ and K_- are the mobilities of positive and negative ions, respectively.

Set (1) is solved subject to the following boundary conditions:

$$I(0) = 0, \qquad I(1) = f \tag{2}$$

Set (1) together with boundary conditions (2) constitutes a boundary problem for a set of ordinary differential equations. As the boundary conditions are prescribed at two points we require, in order to numerically integrate the set, to prescribe at the initial point $x = 0$ the missing boundary condition for the function $\varepsilon(x)$. For each test value of $\varepsilon(0)$ the set is solved, by the Runge–Kutta method, for example, and the value of the function I(x) obtained at the other end of the segment is compared with the prescribed boundary condition. The processes are repeated until $|I(1) - f| < \delta f$, where δ is a previously given small number.

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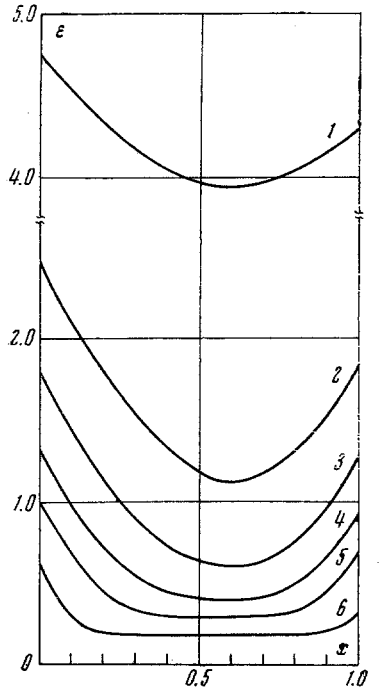

Fig. 1

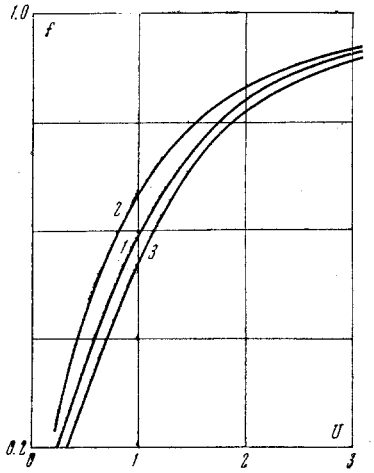

Fig. 2

In the case under investigation this sort of ranging method requires a large amount of machine time. This comes about because the solution is very sensitive to small changes of $\varepsilon(0)$, and $\varepsilon(0)$ must accordingly be sought with quite a high degree of accuracy. If this is not done, the solution enters a region not corresponding to the physical content of the problem. A considerable simplification is achieved by reducing the two-point boundary problem (1), (2) to a Cauchy problem. This technique has been used with success in the numerical solution of various boundary problems in hydrodynamics [3].

We transform set (1), (2) to an initial-value problem by means of a single-parameter transformation of the form

$$\varepsilon = A^{\alpha_1}\varepsilon_1, \quad x = A^{\alpha_3}x_1, \quad I = A^{\alpha_2}I_1, \quad f = A^{\alpha_2}f_1 \tag{3}$$

where A is the parameter of the transformation and α_1, α_2, α_3 are constants which must be determined. In the new variables set (1) acquires the form

$$\begin{aligned} A^{\alpha_1-\alpha_3}\frac{d\varepsilon_1}{dx_1} &= -A^{\alpha_2-\alpha_1}\frac{4\pi}{\varepsilon_1}[f_1-(1+\mu)I_1] \\ A^{\alpha_2-\alpha_3}\frac{dI_1}{dx_1} &= 1-A^{2(\alpha_2-\alpha_1)}\frac{\lambda}{\varepsilon_1^2}I_1(f_1-I_1) \end{aligned} \tag{4}$$

The requirement that set (1) be invariant under the above group of transformations leads to the following equalities for α_1, α_2, and α_3:

$$\alpha_1-\alpha_3=\alpha_2-\alpha_1, \quad \alpha_2-\alpha_3=0, \quad 2(\alpha_2-\alpha_1)=0$$

This implies that $\alpha_1 = \alpha_2 = \alpha_3$. Set (1) then assumes the form

$$\begin{aligned} &\frac{d\varepsilon_1}{dx_1} = -\frac{4\pi}{\varepsilon_1}[f_1-(1+\mu)I_1], \\ &\frac{dI_1}{dx_1} = 1-\frac{\lambda I_1}{\varepsilon_1^2}(f_1-I_1) \quad \text{at} \quad x_1=0,\ I_1(0)=0 \end{aligned} \tag{5}$$

In order to obtain the missing condition we set, for example, $\varepsilon(0) = A$, when $A^{\alpha_1}\varepsilon_1(0) = A$. This boundary condition does not depend on α_1 or A if $\alpha_1 = 1$. The boundary conditions for set (5) are finally written

$$I_1(0)=0, \quad \varepsilon_1(0)=1 \tag{6}$$

Set (5) together with conditions (6) thus constitutes the equivalent Cauchy problem. The value of the parameter A can be found from the boundary condition at the other end of the segment, which in the new variables has the form

$$I_1(x_1 = A^{-1}) = f_1 \tag{7}$$

The parameter A is determined by numerical solution of the Cauchy problem (5), (6) (for a fixed f_1) allowing for condition (7). Knowing A, we find with the aid of (3) the solution of the initial problem for

$$f = Af_1 \tag{8}$$

By varying f_1 it is possible, in principle, to obtain the solution for any f. In other words, reducing the initial problem to a Cauchy problem in the present case is justified when one needs to obtain a solution for f varying in a certain range. Thus, if in the ranging method a large number of test variants must be computed in order to obtain the solution for each fixed value of f, then now each variant gives the solution of the initial problem for f defined by equality (8).

An analysis of set (5) shows that a solution satisfying condition (7) cannot be found for all f_1. A certain boundary value f_1^* exists such that for $f_1 > f_1^*$ there is no solution of the Cauchy problem which satisfies condition (7).

On the other hand, for $f_1 < f_1^*$ a solution can be obtained for any values of the coefficient f however small, although in this case the parameter f_1 must be prescribed with sufficiently high accuracy. For example, in order to obtain the solution of the initial problem for $f \approx 0.3$, the parameter f_1 must be prescribed correct to the seventh decimal place. This sort of computational procedure can be effected on the Minsk-22 digital computer, for example, using the standard Runge–Kutta program. If a numerical solution is required for $f < 0.3$ one can go over to the Runge–Kutta algorithm with doubled accuracy (for the Minsk-22 this means that one can work with numbers having 16 significant figures).

The results of a numerical integration of set (1) are presented in Figs. 1 and 2. Figure 1 shows plots of the electric field versus the coordinate x for various values of the collection efficiency f ($\mu = 0.65$, $\lambda = 4.33$). Curves 1-6 correspond respectively to $f = 0.952$, 0.750, 0.550, 0.405, 0.303, and 0.181. It can be seen that for $f = 0.181$, for example, the curve can be approximated by a broken line varying according to a linear law near the electrodes. Integrating the functions $\varepsilon(x)$ for the various f gives the current-voltage characteristic in the form of the dependence of the collection efficiency on the dimensionless voltage

$$U = \frac{V}{d^2}\sqrt{K_+/eq}$$

Figure 2 shows the current-voltage characteristic calculated in this manner (curve 1) and also, for comparison, the current-voltage characteristics obtained using Boag's formula [4] (curve 2)

$$f = 2[1 + (1 + 2\lambda / 3U^2)^{1/2}]^{-1} \tag{9}$$

and the approximate formula of Vol'f and Polikanov [2] (curve 3)

$$f = \frac{U^2}{2\lambda B}[-1 + \sqrt{1 + 4\lambda B U^{-2}}] \qquad \left(B = \left(\frac{\mu}{(1+\mu)^2} + \frac{U^2}{\lambda}\right)\left(1 + \frac{6U^2}{\lambda}\right)^{-1}\right) \tag{10}$$

(correcting for the misprint in formula (24) of [2]). It can be seen from Fig. 2 that, at any rate for $\mu = 0.65$ and $\lambda = 4.33$ Boag's characteristic (9) passes above the calculated curve, the maximum relative deviation (relative to the calculated curve) for the interval $0.181 < f < 1$ being about 30%. Formula (10), on the other hand, gives results that are too low, although it repeats the shape of the calculated curve rather better in the given range for f (maximum deviation relative to the calculated curve about 24%).

In the case of electron–ion conductivity, it was effectively not possible to obtain a solution for f significantly different from unity upon numerical integration with the doubled-accuracy program.

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